LEARNING MODULE DESCRIPTION

GENERAL INFORMATION

1. Module title: Fourier Series and integrals
2. Term: summer semester
3. Duration: 30 hour of lectures + 30 hours of exercises
4. ECTS: 6
5. Module lecturer: prof. dr hab Leszek Skrzypczak
6. E-mail: lskrzyp@amu.edu.pl
7. Language: English

DETAILED INFORMATION

1. Module aim (aims)
   - The aim of the module is to present the basic and the most important facts concerning the Fourier series of periodic function and Fourier transform defined on real line. Finishing the course students will know the basic concepts of the theory of Fourier series and Fourier transform of functions of one-variable. They can used Fourier series and transform to solve different mathematical problems as well as some problems arising in natural sciences.

2. Pre-requisites in terms of knowledge, skills and social competences (where relevant) Good knowledge in undergraduate mathematical analysis (function series, properties of differentiable functions, elementary properties of analytic functions and Riemann integral).

READING LIST

1. E. Stein, R. Shakarchi, Fourier analysis. An introduction. Princeton University Press 2003.
2. L. Schwartz, Mathematics for the Physical Sciences , Dover Books on Mathematics 2008,
3. R.E. Edwards, Fourier Series. A modern introduction. Springer Verlag 1979.

SYLLABUS:

Week 1: The space of integrable functions L_1. The Hölder and Minkowski inequalities.
Week 2: Convolution of functions defined on real line and one dimensional torus. Young's inequalities. Regularization
Week 3: Fourier coefficients of periodic functions (definition and properties). Partial sums of Fourie series in terms of the convolution.
Week 4: Cesaro's method of summation. Abel's method of summation. Fejer's theorem.
Week 5: The Fourier series and orthogonality. Square integrable functions. The Parseval identity.
Week 6: Pointwise convergence of Fourier series (the Lipschitz condition, the Dini condition, the Jordan condition).
Week 7: Continuous functions with divergence Fourier series. Fourier series versus trigonometric series.
Week 8: The space of Schwartz rapidly decreasing functions on real line and its properties (convolution, pointwise multiplication)
Week 9: The Fourier transform of Schwartz rapidly decreasing functions (inversion formula, Parseval identity, Fourier transform and differentiation).
Week 10: The Fourier transform of integrable functions (Riemann's lemma, Fourier transform of the convolutions).
Week 11: The Fourier transform of other type of functions (square integrable functions, continuous functions) The distributional approach to the definition of the Fourier transform.
Week 12: Poisson's summation formula. Heisenberg's uncertainty principle.
Week 13: Supports of continuous and integrable functions. Fourier transform, smoothness and compact support. The Paley-Wiener theorem
Week 14: Applications of the Fourier transform and series (the wave equation, the heat equation, isoperimetric inequalities)
Week 15: Remarks on Fourier transform of multivariable functions. Fourier transform and radiality.