Module Title: Introduction to wavelets

Pre-requisites in terms of knowledge, skills and social competences (where relevant)

It is assume that the student has knowledge congruent with the traditional course of mathematical analysis of functions of one and several real variables.

Module aim (aims)

Wavelet analysis was created in 70-ties last century as a method of expansion of functions in to series. It is an alternative method to the classical method of representation functions via Fourier series. Wavelet expansions are the expansion with respect to the family of functions, that is constructed from a single function via translations and dilatations. Functions that give us a ``good'' expansions constructed in that way are called wavelets. The wavelets theory quickly appeared very useful tool in many applications in mathematics, physics and computer science (e.g. signal processing, image compression). The aim of the module is to introduce the mathematical background of the wavelets.

Syllabus

1. Convergence and bases in Banach spaces; unconditional convergence, absolute convergence conditional convergence.
2. Orthogonal and orthonormal bases bases in Hilbert spaces. Frames.
3. The Fourier transform and Fourier series of integrable functions – the Riemann-Lebesgue Lemma, Parseval identity, convolution of functions.
4. The Haar system and Stromberg system – the simplest wavelet systems.
5. Multi-scale analysis for function of one variable –construction of the wavelet
6. Mayer wavelets
7. Wavelets of spline functions
8. Daubechies wavelets
9. Wavelets system in several dimensions
10. Wavelet and smoothness of the functions
11. Convergence of wavelet expansions (uniform, pointwise, a.e.)

Reading List

1. P. Wojtaszczyk, A Mathematical introduction to wavelet theory, Cambridge University Press 1997
2. E. Hernandez, G.Weiss, A first Course on Wavelets, CRC Press, 1996
3. Y. Meyer, Wavelets and operators, Cambridge University Press 1992
4. A. Cohen, Numerical Analysis of wavelets methods, North-Holland Elsevier 2003
5. I. Daubechies, Ten lectures on wavelets, SIAM 1992