LEARNING MODULE DESCRIPTION (SYLLABUS)

I. General information

1. Harmonic Analysis
2. Module code DAHA UMS
3. Module type – optional
4. Programme title: mathematics
5. 2nd cycle of studies and PhD studies
6. Year of studies (where relevant) 1 and 2
7. Terms in which taught (summer/winter term) summer
8. Lectures 30 hours; practical classes: 30 hours)
9. Number of ECTS credits 6
10. Prof. dr hab. Leszek Skrzypczak / e-mail: lskrzyp@amu.edu.pl
11. Language of classes: English

II. Detailed information

1. Module aim (aims)
   The aim of the module is the presentation of main ideas of harmonic analysis. It will be described how the ideas are realized for functions and distributions defined on Euclidean spaces $\R^d$. The main concepts of the harmonic analysis on $\R^d$ will defined, the relation among them will described as well as their applications.

2. Pre-requisites in terms of knowledge, skills and social competences (where relevant)
   Analysis - main course,
3. Module learning outcomes in terms of knowledge, skills and social competences and their reference to programme learning outcomes

Learning outcomes symbol*	Upon completion of the course, the student will:	Reference to programme learning outcomes#
E_01	Student can use the spacer of functions integrable with p-power and their properties to solve the problems of harmonic analysis	KMAT2_U11, KMAT2_W11, KMAT2_W07
E_02	Student knowi and can used the concept of tempered distribution. He/She can calculate derivatives, Fourier transforms and convolutions of particular distributions	KMAT2_U10, KMAT2_W06, KMAT2_W07
E_03	Student knows the relation between Fourier transform and smoothness of functions and distributions. She/He is able to use the Fourier transform for defining the smoothness of function and distributions.	KMAT2_U10, KMAT2_U12, KMAT2_W07
E_04	Student can use the Fourier transform and convolution to solve some PDE.	KMAT2_U03, KMAT2_U06, KMAT2_W03
E_05	Student knows the basic types of operators of harmonic analysis (e.g. singular integrals, Fourier multipliers)	KMAT2_U14, KMAT2_W07

	and can check their continuity in simpler cases.	

** module code, e.g. KHT_01 (KHT – module code in USOS; stands for Polish “Kataliza Heterogeniczna” /Heterogeneous Catalysis/)*
[#] programme learning outcomes (e.g. K_W01, K_U01, …); first K stands for programme title symbol in Polish, W for “wiedza” (knowledge) in Polish, U – for “umiejętności” (skills) in Polish, K – for “kompetencje społeczne” (social competences) in Polish
01, 02… - learning outcome number

4. Learning content

Module title		
Learning content symbol*	Learning content description	Reference to module learning outcomes [#]
TK_01	*Spaces of functions integrable with p-power – completness of the spacer, dual spacer, operators of strong and weak types, the Riesz-Thorin intrepolation Theorem, Marcinkiewicz interpolation Theorem, convolutions, regularization.*	E_01
TK_02	*Fourier transform of integrable functions – elementary properties, the Riemann-Lebesgue Lemma.*	E_01, E_05
TK_03	*Hardy-Littlewood maxima function, Calderon-Zygmund decomposition, information about Hardy spaces and BMO*	E_01, E_05
TK_04	*Schwartz spacer of rapidly decreasing functions, tempered distributions and their Fourier transform, inversion formula for Fourier transform, convolution of distributions.*	E_02
TK_05	*The Fourier transform of functions from $L^p(\R^d)$ - the Plancherela theorem, Hausdorfa-Younga inequality and the Hilbert transform.*	E_01, E_02
TK_06	*The Fourier trans form and smoothness and the compactness of the support – the Paley-Wiener Theorem and the Paley-Wiener-Schwartz theorem.*	E_03
TK_07	*Singular integral operators: almost orthogonal, the Calderona-Zygmunda operators.*	E_05
TK_08	*Littlewood-Paley Theorem and quadratic functions,*	E_01, E_05
TK_09	*Fourier multipliers - the H\"ormander theorem and the Feffermann theorem.*	E_05
TK_10	*Applications to PDE and to wavelets .*	E_04

** e.g. TK_01, TK_02, … (TK stands for “treści kształcenia” /learning content/ in Polish)*
[#] e.g. KHT_01 – module code as in Table in II.3

5. Reading list

- J. Duoandikoetxea, Fourier Analysis, AMS 2001.

- W. Rudin, Analiza funkcjonalna, PWN 2001.
- L.Schwartz, Metody matematycne w fizyce, PWN 1984.
- E.Stein, R.Shakarchi, Functional Analysis. Introduction to further topics in Analysis. Princeton Univ. Press 2011.

- E.Stein, R.Shakarchi, Fourier Analysis. An introduction. Princeton Univ. Press 2003.

6. Information on the use of blended-learning (if relevant)
7. Information on where to find course materials- faculty library, internet.

III. Additional information

1. Reference of learning outcomes and learning content to teaching and learning methods and assessment methods

Module title			
Symbol of module learning outcome*	Symbol of module learning content#	Methods of teaching and learning	Assessment methods of LO achievement&
E_01	TK_01, TK_02, TK_03, TK_05, TK_08	Lectures and practical classes	Written and oral exam
E_02	TK_04, TK_05, TK_08	Lectures and practical classes	Written and oral exam
E_03	TK_06	Lectures and practical classes	Written and oral exam
E_04	TK_10	Lectures and practical classes	Written and oral exam
E_05	TK_02, TK_03, TK_07, TK_09	Lectures and practical classes	Written and oral exam

** e.g. KHT_01 – module code as in Table in II.3 and II.4*
 # e.g. TK_01 – learning content symbol as in II.4
& Please include both formative (F) and summative (S) assessment

It is advisable to include assessment tasks (questions).

2. Student workload (ECTS credits)

Module title:	
Activity types	Mean number of hours* spent on each activity type
Contact hours with the teacher as specified in the programme	30 hours of lectures and 30 hours of exercises
Solving home exercises	25
Book study	25
Preparation to classes	20
Exam preparation	20
Total hours	150
Total ECTS credits for the module	6

** Class hours – 1 hour means 45 minutes*
#Independent study – examples of activity types: (1) preparation for classes, (2) data analysis, (3) library-based work, (4)writing a class report, (5) exam preparation, etc.

3. Assessment criteria: good understanding of the presented concepts, of relation among them, of presented theorems and ability of application of the understanding in solving simple problems

4. Titles of classes

Syllabus:	
Week 1	*Spaces of functions integrable with p-power – completness of the spacer, dual spacer, , convolutions, regularization*
Week 2	*Operators of strong and weak types, the Riesz-Thorin intrepolation Theorem, Marcinkiewicz interpolation Theorem*
Week 3	*Fourier transform of integrable functions –elementary properties, the Riemann-Lebesgue Lemma.*
Week 4	*Hardy-Littlewood maxima function, Calderon-Zygmund decomposition, information about Hardy spaces and BMO*
Week 5	*Schwartz spacer of rapidly decreasing functions, tempered distributions*
Week 6	*Fourier transform of tempered distributions , inversion formula for Fourier transform, convolution of distributions.*
Week 7	*The Fourier transform of functions from $L^p(\R^d)$ - the Plancherela theorem, Hausdorfa-Younga inequality*
Week 8	*The Hilbert transform.*
Week 9	*The Fourier trans form and smoothness and the compactness of the support – the Paley-Wiener Theorem and the Paley-Wiener-Schwartz theorem.*
Week 10	*The Fourier trans form and smoothness and the compactness of the support – the Paley-Wiener Theorem and the Paley-Wiener-Schwartz theorem part lskrzypII*
Week 11	*Singular integral operators: almost orthogonal, the Calderona-Zygmunda operators.*
Week 12	*Singular integral operators: almost orthogonal, the Calderona-Zygmunda operators. partII*
Week 13	*Littlewood-Paley Theorem and quadratic functions,*
Week 14	*Fourier multipliers - the H\"ormander theorem and the Feffermann theorem.*

Week 15	*Zastosowania: do równań różniczkowych cząstkowych, konstrukcje falkowe.*